

**INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE**  
**B.MATH - Second Year, First Semester, 2023-24**  
**Statistics - II, Backpaper Examination**

**Time: 3 hours**

**Maximum Marks: 50**

1. Let  $X_1, \dots, X_n$  be independent random variables with densities

$$f_{X_i}(x_i|\theta) = \begin{cases} \frac{1}{2^i\theta} & \text{if } -i(\theta - 1) < x_i < i(\theta + 1); \\ 0 & \text{otherwise,} \end{cases}$$

$1 \leq i \leq n$ , where  $\theta > 0$ .

(a) Find a two-dimensional sufficient statistic for  $\theta$ .

(b) Find the maximum likelihood estimator of  $\theta$ . [7+8]

2. Consider a random sample from  $N(0, \sigma^2)$ .

(a) Find the UMVUE of  $\sigma$ .

(b) Show that the UMVUE of  $\sigma$  is a consistent estimator.

(c) Find the asymptotic distribution of the UMVUE of  $\sigma$ . [5+3+4]

3. Suppose  $X_1, X_2, \dots, X_n$  is a random sample from Poisson( $\lambda$ ). Consider testing

$$H_0 : \lambda \leq 1 \text{ versus } H_1 : \lambda > 1.$$

(a) Show that the conditions required for the existence of a UMP test are satisfied here.

(b) Derive the UMP test of level  $\alpha$ . [3+5]

4. A large shipment of parts is received, out of which 5 are tested for defects. Let  $X$  denote the number of defective parts in the sample, and  $\theta$  be the proportion of defective parts in the population. From past shipments it is known that  $\theta$  has a Beta(1, 9) distribution.

(a) Find the HPD estimate of  $\theta$  if  $x = 0$  is observed.

(b) Find a 95% credible set for  $\theta$  if  $x = 0$  is observed.

(c) For testing  $H_0 : \theta \leq 0.10$  versus  $H_1 : \theta > 0.10$ , find the posterior odds ratio. [5+5+5]